Multiple concurrent recursive least squares identification (MCRLS ID)

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MCRLS ID

- Multiple concurrent recursive least squares identification
- New algorithm for parameter estimation
- Use when governing equations do not allow manipulation into linear form
- Approximation to non-linear ID
  - Very efficient – computation, code size
  - Optimized for on-line application
  - Extensible
- Works by segmenting nonlinear problem into multiple linear problems, concurrently combining results during execution
- Developed for spacecraft mass-property ID application (paper in this conference). Being applied for space flight test on embedded DSP on experimental spacecraft.

Outline

- Introduction
  - Parameter ID, RLS, spacecraft example, related work
- Simple example
- General algorithm
  - Benefits
  - Extensions
  - Convergence
  - Optimality
  - Viability
- Spacecraft example
- Conclusions

Introduction

- Parameter estimation, linear form:
  - Cast governing equations into form $\boldsymbol{Ax} = \boldsymbol{b}+\varepsilon$
  - $\boldsymbol{A}$ known perfectly; unknown parameters appear (linearly) in $\boldsymbol{x}$; $\boldsymbol{b}$ noise-free; noise in ε
  - Solution minimizes sum squared error: $\hat{x} = \left(A^T W A\right)^{-1} A^T W b$
  - Equivalent recursive algorithm (RLS) exists
- Challenge is in manipulating governing equations into "regression form": $\boldsymbol{Ax} = \boldsymbol{b}+\varepsilon$
- Spacecraft example: $I^{-1}((L \times D)B(F)T_k - \omega \times (I\omega)) = \dot{\omega}$
- Multiple sets of parameters, appear nonlinearly
- Related work: Tanygin 1997, Bergmann 1987, Wilson 1994

MCRLS – simple example

- Governing equation: $a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2 = b$
- Known values, a_1, a_2, a_{12}; measurement, b; unknowns x_1, x_2.
- Cannot be put directly into regression form
- One feasible approach that is for a problem of this simplicity is to let $A = [a_1 \quad a_2 \quad a_{12}]$; $x = [x_1 \quad x_2 \quad x_1 x_2]^T$
- MCRLS approach: segment into 2 linear parts:
  - x_1 ID: $(a_1 + a_{12}\hat{x}_2)x_1 = b - a_2\hat{x}_2$
  - x_2 ID: $(a_2 + a_{12}\hat{x}_1)x_2 = b - a_1\hat{x}_1$
- Approximation error depends on difference between true and ID'ed values used to ID other parameters.

Real example (center of mass ID)

Equations of motion for a thruster-controlled spacecraft:

$$\dot{\omega} = I^{-1}((L \times D)B(F_{nom} + F_{bias} + F_{random,k})T_k + \tau_{disturb} - \omega \times (I\omega))$$

Manipulated EOM:

$$C \equiv C_{nom} + \Delta; L = L_{nom} - \Delta\begin{bmatrix}1 & 1 & \ldots & 1\end{bmatrix}$$

$$\hat{I}^{-1}\begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}_k \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \dot{\omega} + \hat{I}^{-1}(\omega \times (\hat{I}\omega)) - \hat{I}^{-1}(L_{nom} \times D)(F_{nom} + \hat{F}_{bias})T_k$$

$$c_k \equiv D(F_{nom} + \hat{F}_{bias})T_k$$

LS (or RLS) formulation: $A_k x = b_k$

$$A_k = I^{-1}\begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}_k ; x = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} ; b_k = \dot{\omega} + I^{-1}(\omega \times (I\omega)) - I^{-1}(L_{nom} \times D)F_{nom}T_k$$

MCRLS – General algorithm

- Divide vector of unknown parameters into arbitrarily large number of groups, each containing arbitrarily large number of parameters. Preferred is to maintain as much linearity as possible (fewer, bigger groups).
- Develop (linear) regression equation for each group – *if possible*
- Run each RLS ID, sharing results with other IDs.
- Appropriate initialization of covariance matrices, weighting are especially important here.

$$\text{Regression Eqn for } \hat{x}_1\text{: } A_1(\hat{x}_2,\cdots,\hat{x}_n,\cdots)x_1 = b_1(\hat{x}_2,\cdots,\hat{x}_n,\cdots)$$

$$\text{Regression Eqn for } \hat{x}_2\text{: } A_2(\hat{x}_1,\hat{x}_3,\cdots,\hat{x}_n,\cdots)x_2 = b_2(\hat{x}_1,\hat{x}_3,\cdots,\hat{x}_n,\cdots)$$

$$\vdots$$

$$\text{Regression Eqn for } \hat{x}_n\text{: } A_n(\hat{x}_1,\cdots,\hat{x}_{n-1},\cdots)x_n = b_n(\hat{x}_1,\cdots,\hat{x}_{n-1},\cdots)$$

Benefits

- Enables approximate solution of nonlinear ID using linear methods
- Simple to add/remove parameters from the list to be ID'ed
- Segmentation enables controlled updating based on physical reasoning
- Uses RLS algorithm – very fast and compact. Re-uses for each ID, saving on code size, time and cost of development, testing, and sustaining engineering

Extensions

- RLS is well suited, but others could be used
- On-line and off-line. Repeated cycling off-line reduces dependence on accuracy of initialization.
- Incorporation of known variation – e.g., nominal fuel mass change. IDs are designed to ID the deviation from nominal (which includes the “known variations”)

Convergence, Optimality

- No convergence proof
- Testing indicates that with sufficiently accurate initial estimates, ID accuracy is comparable to that with true vs. estimated parameters used.
- With careful estimate-error-covariance-matrix initialization and measurement weighting, MCRLS ID can only approach the mathematical optimality of a Kalman Filter or the results of a nonlinear optimization
- But in virtually all real applications, strict requirements of the LS formulation are not met either.
- Algorithmic sub-optimality is only important as compared to other sub-optimal effects (un-modeled disturbances or dynamics, unknown variations in “known” parameters, dropped higher order terms, sensor biases, etc.)
- At some point, additional algorithmic complexity is not warranted due to the deviations from the idealized problem statement. (e.g., the *A* matrix in standard system ID)

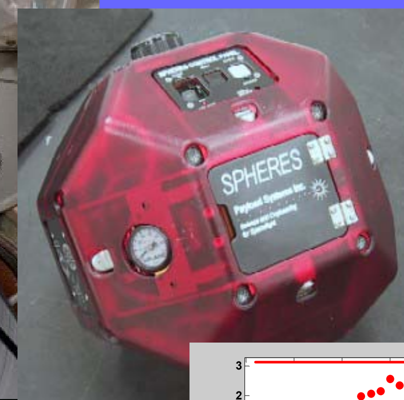
Viability, tests

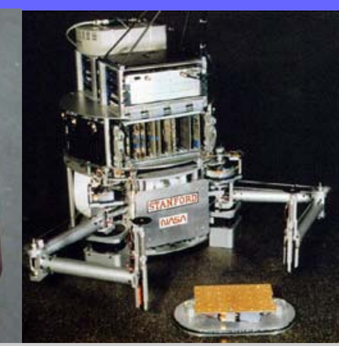
- Not universal
- Must be able to put each parameter group into linear regression form
- Test 1 (if MCRLS approximation error is negligible in the presence of other un-modeled effects): simulation – test ID with standard MCRLS vs. using true values
- Test 2 (determines how much better an optimal nonlinear ID would perform): off-line nonlinear ID, compare accuracy
- Analytical tests may be possible

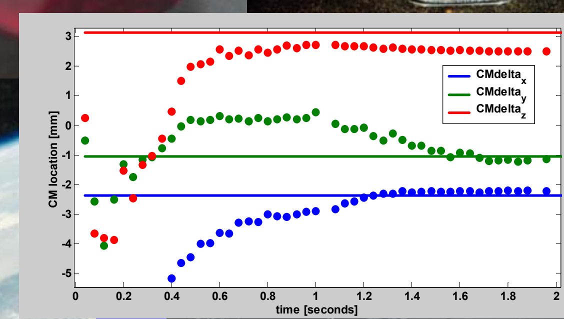
Research objective: For thruster-controlled spacecraft, **identify mass- and thruster- properties** (I, CM, …) using gyros, under normal vehicle control. Applicable for advanced control, estimation, or FDI. Develop and validate through application on realistic simulations and hardware. Flight test on SPHERES aboard ISS in 2005.

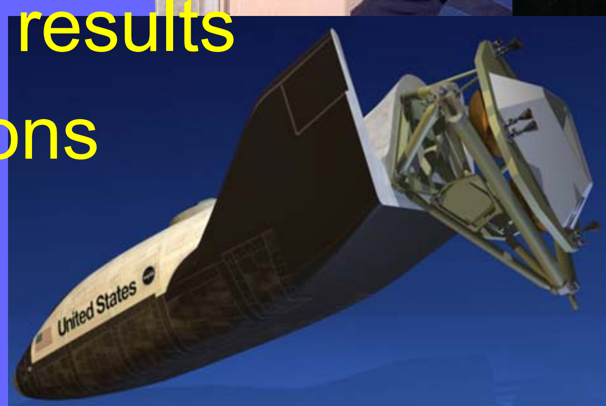
Outline:

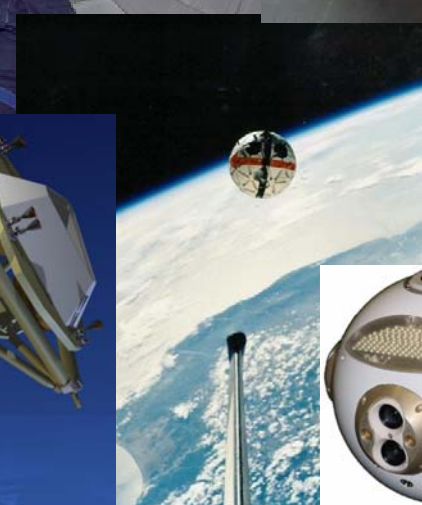
- Spacecraft ID
- MCRLS
- SPHERES
- KC-135A results
- Conclusions

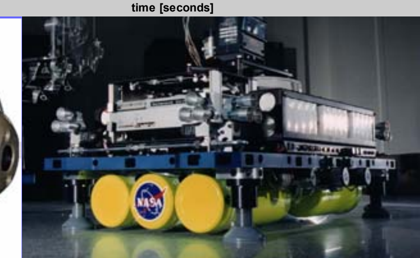












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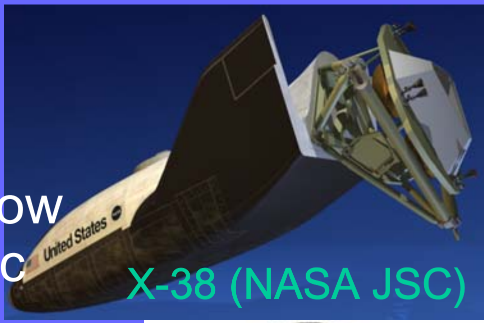
Spacecraft on-line identification (ID)

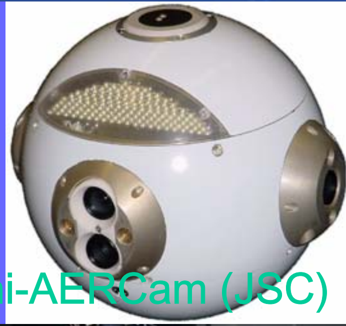
- **Why** we want to ID mass and thruster properties on-line:
  - Advanced control, estimation, fault detection and isolation (FDI)
  - Fuel consumption, variable payload, variable configuration
  - Small disturbances in space → accurate knowledge is important
- **ID Technologies** for thruster-controlled spacecraft:
  - Small disturbances in space → motion-based (**gyros** are sufficient) analysis is possible, and often more accurate than ground testing
  - **ID:** center of mass (CM) location; inertia matrix; inverse inertia matrix; thruster strength; total mass. Gyros only, gyros+accels (or other sensors).
  - Generic technologies developed through application to specific problem statements provided by NASA JSC for X-38 and Mini-AERCam
  - Software-only solution (uses existing sensors)
  - Applicable to a broad class of spacecraft
- **Implementing for SPHERES** – to fly on ISS in 2005

Mass- and thruster-property ID

- Problem statement:
  - For a thruster-controlled spacecraft, with relatively low rotation rates, realistic sensor noise models, realistic thrust variability, using gyros only
  - ID mass and thruster properties
- Principal challenge:
  - Unknown parameters do not appear linearly in the equations of motion ➔ direct Least Squares (LS) solution not possible
- Related Research:
  - Tanygin and Williams (1997) – spinning, coasting, LS
  - Bergmann *et al* (1987) – Second order filter
  - Wilson and Rock (1994) – RLS combined thruster/mass ID; used for on-line neural-network control reconfiguration following multiple thruster failures



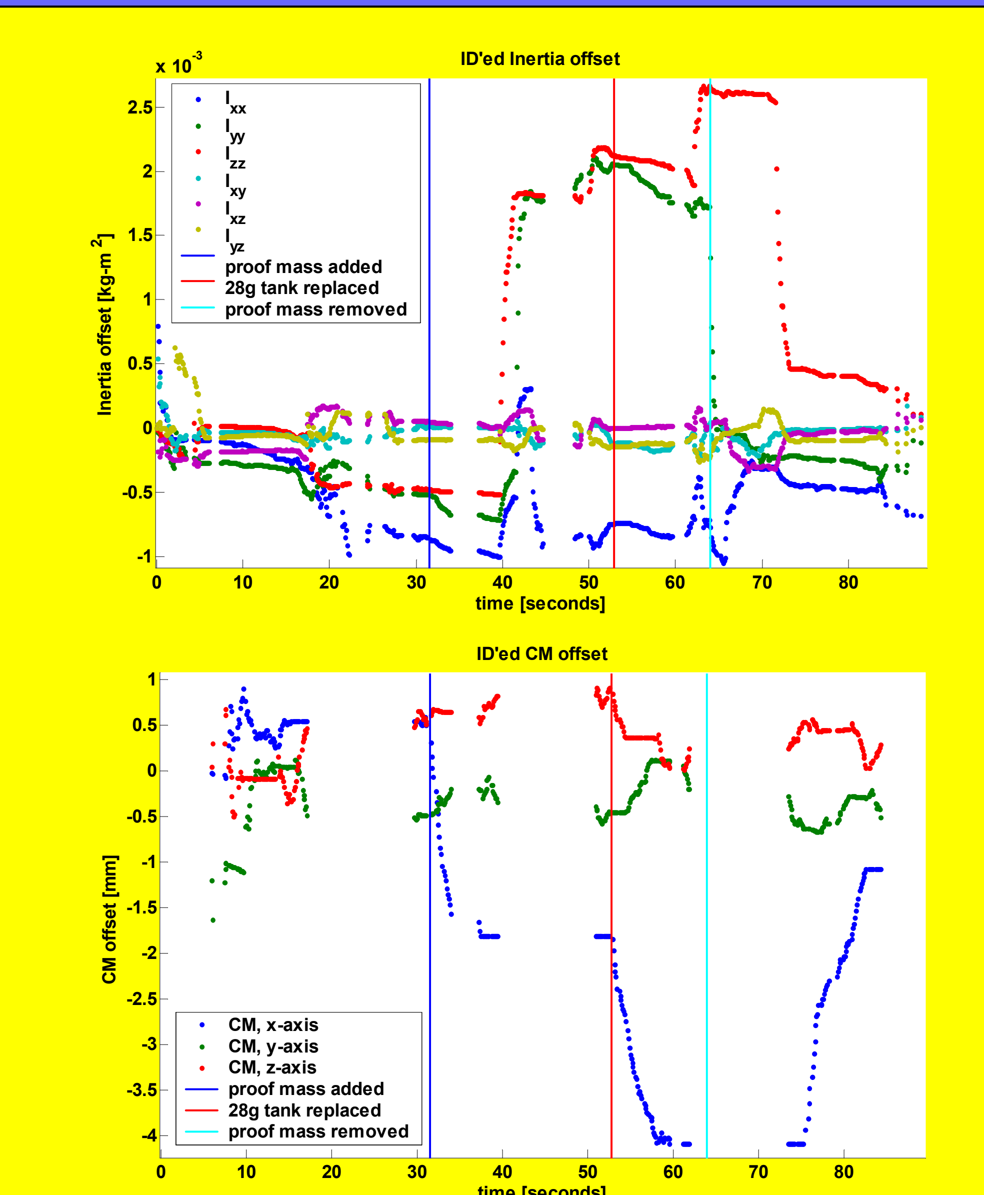
X-38 (NASA JSC)


Mini-AERCam (JSC)

S4 (NASA Ames)

SPHERES (MIT SSL)

Inertia and CM ID results



- Proof mass added, removed, near-empty tank replaced
- Fast response indicates feasibility for spacecraft with variable payload or internal reconfiguration
- Certain sections off apparent convergence are actually periods of little excitation
- Meaningful numerical results will require more data, resolution of remaining thruster characterization issues
- So far, effect of fuel slosh appears minimal – as hoped, it averages out and does not impact the ID results

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Conclusions

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- Paper and presentation are available at http://intellization.com/files/

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